Design Approach of Improved Response DC Motor Analyzing Motor Characteristics in terms of Electrical and Mechanical Constants with Control Engineering Parameters

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Abstract—Accuracy of adjustment of different parameters of DC motor makes the motor speed very smooth and steady, is the main prerequisite for designing a perfect DC motor so that it can be used for very precise rotation. So, the main aim of this paper is to provide a good guidance for some design considerations how to improve the response of DC motor minimizing its overshoot (%OS), rise time (tr), settling time (ts), steady state time (tss) and increasing the steady state speed until unity supplying step input to the motor. To observe the performance of the motor, equation of the transfer function has been derived setting all initial conditions to zero. Two machine constant coefficients (Motor torque constant (Kt) and back EMF constant (Ke)) have been investigated to derive the compact form related to the motor size and material property. The material property relates the electrical constants, armature resistance (Ra) and armature inductance (La). A mathematical approach, based on electromagnetic field theory, has been used to formulate the two machine constants. The system has been simulated using MATLAB to analyze the different parameters of DC motor that how the speed of the motor is improved so that the range of parameter's value can be specified.

Key Words: - DC Motor, Transfer Function, Parameter Adjustment, Design Approach, Speed.

1 INTRODUCTION

The DC motor has been popular in the industry control area for a long time even though its maintenance costs are higher than the induction motor [1], because it has many good characteristics, for example, high starting torque, high response performance and easier to be linear control [2][3].And it is widely used in speed control systems which need high control requirements, such as rolling mill, double-hulled tanker, high precision digital tools, etc.

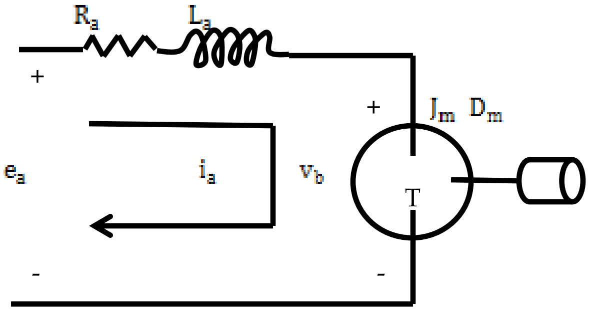
Because of high importance in control scheme for controlling speed of motor, different kinds of controller have been developed to control the speed of motor properly in accordance with needs, for example, PID parameters have been selected using particle Swarm Optimization (PSO) method for designing a PID speed controller to control the speed of DC motor [4], a real-time DC motor PID speed controller has been designed using an eight-bit C505C-L microcontroller [5].

But this work presents an assumption of values of different parameters, to design a motor model at the time of constructing it, for which the speed of motor can be achieved quickly at steady position with less overshoot, settling time, steady state time and rise time for maximum speed.

The effect major parameters related to motor, armature resistance (Ra), armature inductance (La), motor torque constant (Kt), back EMF constant (Ke), equivalent viscous damping coefficient (Dm), equivalent inertia coefficient (Jm), electrical time constant (te), mechanical time constant (tm), have been simulated using MATLAB program to assume the ideal values of the parameters so that almost precise speed motor can be constructed without using controllers. The following sections will deal the work in details.

2 MATHEMATICAL MODELING

In the analysis of DC motors, the equations for the motor indicate the presence of two time constants; these are related to transfer function of the motor. One is a mechanical time constant and the other is an electrical time constant. Also these two time constants are related to armature resistance and inductance, motor torque constant, back EMF constant and equivalent inertia constant. The equivalent circuit diagram and equations are derived below [6].



Where,
$e_a(t)$= Applied armature voltage (Volts)
$i_a(t)$= Applied armature current (Amps)
Jm= Equivalent inertia constant (Kg-m^2)
Dm= Equivalent viscous damping constant (N- m-s/rad)
Kt= Motor torque constant (N-m/A)
Ke= Motor back EMF constant (V/rad/sec)
V_m= Motor velocity (rad/sec)

T= Motor torque (N-m)
Ra= Armature resistance (Ω)
La= Armature inductance (Ω)
v_b= Back EMF of motor (volts)

According to KVL-

$$e_a = R_a i_a + SL_a i_a + v_b$$
$$e_a = (R_a + SL_a) i_a + v_b \tag{1}$$

Since the current-carrying armature is rotating in the magnetic field, its voltage is proportional to speed. Thus, for back EMF of motor is-

$$v_b = K_e V_m \tag{2}$$

So the equation is-

$$e_a = (R_a + SL_a) i_a + K_e V_m$$
$$e_a = \left(1 + S\frac{L_a}{R_a}\right) i_a R_a + K_e V_m \tag{3}$$

The torque is developed by the motor is proportional to the armature current; thus,

$$T = K_t i_a \tag{4}$$

Where T is the torque developed by the motor, and K_t is the constant of proportionality, called the motor torque constant, which depends on the motor and magnetic field characteristics. In a consistent set of units, the value of K_t is equal to the value of K_e.

$$T = SJ_m V_m + D_m V_m \tag{5}$$

$$i_a = \frac{SJ_m V_m + D_m V_m}{K_t} \tag{6}$$

$$e_a = \left(1 + S\frac{L_a}{R_a}\right) \frac{(SJ_m V_m + D_m V_m)}{K_t} R_a + K_e V_m$$

$$\frac{V_m}{e_a} = \frac{\frac{1}{K_e}}{\left\{\frac{R_a(SJ_m + D_m)}{K_t K_e}\right\}\frac{L_a}{R_a} S + \frac{J_m R_a}{K_t K_e} S + \frac{J_m D_m}{K_t K_e} + 1}$$

$$\frac{V_m}{e_a} = \frac{\frac{1}{K_e}}{\left\{\frac{R_a J_m}{K_t K_e}\right\}\frac{L_a}{R_a} S^2 + \left\{\frac{D_m R_a}{K_t K_e}\frac{L_a}{R_a} + \frac{R_a J_m}{K_t K_e}\right\} S + \frac{J_m D_m}{K_t K_e} + 1}$$

$$\frac{V_m}{e_a} = \frac{\frac{1}{K_e}}{t_m t_e S^2 + (t_d t_e + t_m) S + (t_d + 1)} \tag{7}$$

Where the mechanical time constant is

$$t_m = \frac{R_a J_m}{K_t K_e} \tag{8}$$

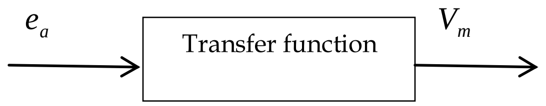
and electrical time constant is

$$t_e = \frac{L_a}{R_a} \tag{9}$$

And the damping factor is

$$\delta = 0.5\sqrt{t_m / t_e} \tag{10}$$

In block diagram, the transfer function can be represented as



3 SIMULATED RESULT

Guidance for choosing data to construct a better response DC motor, transfer function has been simulated varying different parameters and thus the best value of a parameter can be approximated for the design of a motor. In terms of control engineering parameters and supplying step input to the motor, the output response has been observed; these are explained below successively.

3.1 Variation of Armature Resistance (Ra) Only

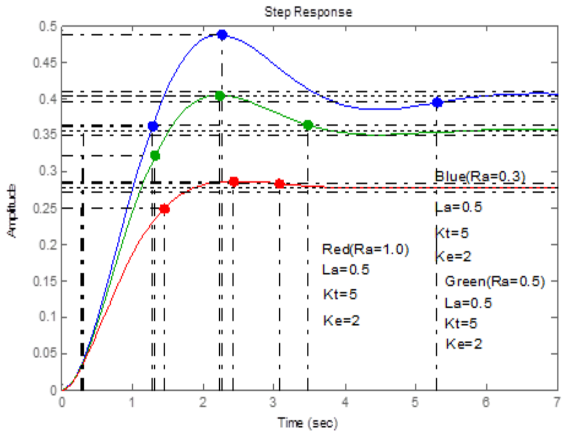


Fig.1. Step response varying armature resistance only

TABLE 1

VALUES OF CONTROLLED PARAMETERS

Obs.	% OS	t_r Rise time (Sec)	t_p Peak Time (Sec)	t_s Settling Time (sec)	t_{ss} Steady State time (sec)	Steady State Output
Blue Curve	21	0.979	2.27	5.31	7	0.42
Green Curve	13.5	1.03	2.24	3.47	7	0.36
Red Curve	3.22	1.19	2.42	3.07	4.5	0.275

TABLE 2

VALUES OF CONSTANTS RELATED TO MOTOR

Obs.	R_a	L_a	t_e	K_t	K_e	D_m	J_m	t_m
Blue Curve	0.3	0.5	1.67	5	2	8	10	0.3
Green Curve	0.5	0.5	1	5	2	8	10	0.5
Red Curve	1.0	0.5	0.5	5	2	8	10	1

From the fig.1, table 1 and table 2, we observe that if armature resistance is increased then overshoot, steady state time and settling time gradually decrease. And in this case the speed of motor decreases for carrying same load. But rise time increases for both cases, though the electrical time constant is deviated from unity. Even, if we increase the armature resistance gradually, once the motor will reach into shut down condition.

3.2 Variation of Armature Inductance (La) Only

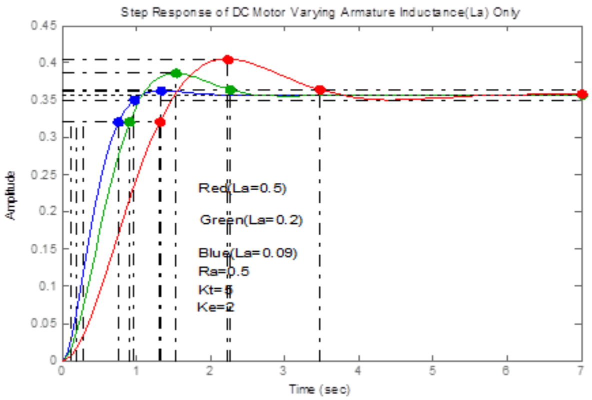


Fig.2. Step response varying armature inductance only

TABLE 3

VALUES OF CONTROLLED PARAMETERS

Obs.	% OS	t_r Rise time (Sec)	t_p Peak Time (Sec)	t_s Settling Time (Sec.)	t_{ss} Steady State time (Sec.)	Steady State Output
Blue Curve	1.39	0.632	1.33	0.964	2.5	0.357
Green Curve	8.14	0.724	1.54	2.26	4	0.357
Red Curve	13.5	1.03	2.24	3.47	7	0.357

TABLE 4

VALUES OF CONSTANTS RELATED TO MOTOR

Obs.	R_a	L_a	t_e	K_t	K_e	D_m	J_m	t_m
Blue Curve	0.5	0.09	0.18	5	2	8	10	0.3
Green Curve	0.5	0.2	0.40	5	2	8	10	0.5
Red Curve	0.5	0.5	1.0	5	2	8	10	0.5

From the fig.2, table 3 and table 4, in variation of armature inductance towards increasing, peak time, settling time, rise time, steady state time all increase. But speed at steady state is the same in all cases. For increasing the armature inductance, overshoot increases until a certain value then decreases but other parameters are caused to problem.

3.3 Variation of Torque Constant (Kt)

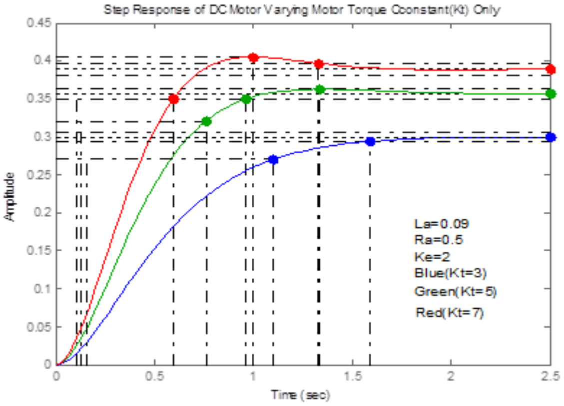


Fig.3. Step response varying torque constant

TABLE 5

VALUES OF CONTROLLED PARAMETRS

Obs.	% OS	t_r Rise time (Sec.)	t_p Peak Time (Sec.)	t_s Settling Time (Sec.)	t_{ss} Steady State time (Sec.)	Steady State Output
Blue Curve	0.005	0.94	>2.5	1.59	2.5	0.3
Green Curve	1.39	0.632	1.33	0.964	2.5	0.357
Red Curve	4.19	0.484	1.0	1.33	2.0	0.389

TABLE 6

VALUES OF CONSTANTS RELATED TO MOTOR

Obs.	R_a	L_a	t_e	K_t	K_e	D_m	J_m	t_m
Blue Curve	0.5	0.09	0.18	3	2	8	10	0.833
Green Curve	0.5	0.09	0.18	5	2	8	10	0.5
Red Curve	0.5	0.09	0.18	7	2	8	10	0.357

From the fig.3, table 5 and table 6, for increasing the value of motor torque constant which is related to motor size and material property, rise time, peak time, steady state time decrease but settling time is fluctuating. In this case steady state output is increased with increasing overshoot.

3.4 Variation of Motor Back EMF Constant (Ke)

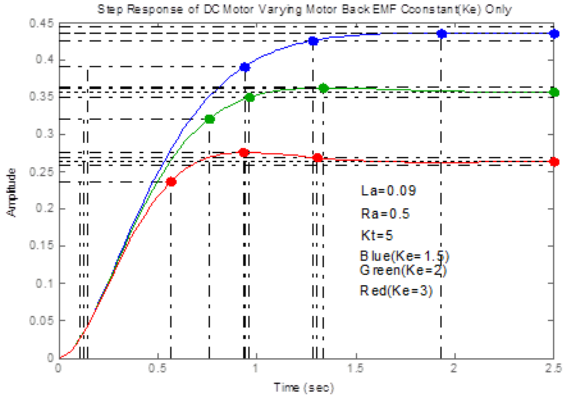


Fig.4. Step response varying motor back EMF constant

TABLE 7

VALUES OF CONTROLLED PARAMETRS

Obs.	% OS	t_r Rise time (Sec.)	t_p Peak Time (Sec.)	t_s Settling Time (Sec.)	t_{ss} Steady State time (Sec.)	Steady State Output
Blue Curve	0.224	0.793	1.93	1.28	2.0	0.435
Green Curve	1.39	0.632	1.33	0.964	2.5	0.375
Red Curve	4.93	0.458	0.934	1.03	2.5	0.263

TABLE 8

VALUES OF CONSTANTS RELATED TO MOTOR

Obs.	R_a	L_a	t_e	K_t	K_e	D_m	J_m	t_m
Blue Curve	0.5	0.09	0.18	5	1.5	8	10	0.667
Green Curve	0.5	0.09	0.18	5	2	8	10	0.5
Red Curve	0.5	0.09	0.18	5	3	8	10	0.333

From the fig.4, table 7 and table 8, if we change the value of motor back EMF constant towards increasing then the rise time, peak time, steady state output decrease. But the settling time decreases with fluctuation of its magnitude with respect to increasing motor back EMF constant. In this case, overshoot and steady state time increase.

Obs.	R_a	L_a	t_e	K_t	K_e	D_m	J_m	t_m
Blue Curve	0.5	0.09	0.18	4	3	8	10	0.417
Green Curve	0.5	0.09	0.18	6	2	8	10	0.417
Red Curve	0.5	0.09	0.18	8	1.5	8	10	0.417

3.5 Varying Torque Constant Increasing but Back EMF Constant Decreasing

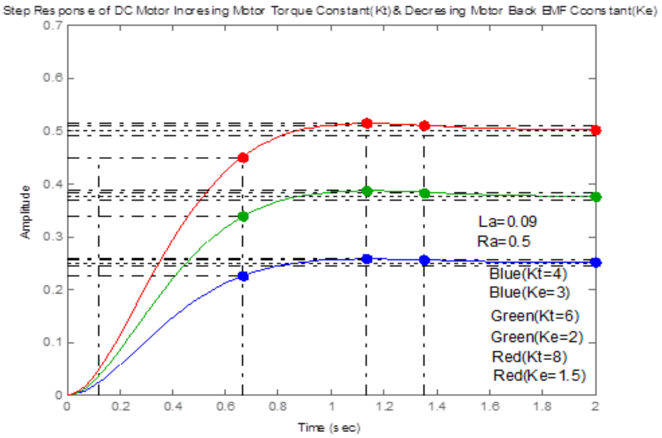


Fig.5. Step response varying Torque Constant Increasing but Back EMF Constant Decreasing

From the fig.5, in case of increasing the torque constant and decreasing back EMF constant such that the multiplication of both constants is same so that mechanical time constant is not change due to changing the constants, overshoot, rise time, peak time, settling time and steady state time are not change. But the speed of motor is increased owing to increasing torque constant though back EMF constant is increasing.

TABLE 9

VALUES OF CONTROLLED PARAMETRS

Obs.	% OS	t_r Rise time (Sec.)	t_p Peak Time (Sec.)	t_s Settling Time (Sec.)	t_{ss} Steady State time (Sec.)	Steady State Output
Blue Curve	2.73	0.546	1.13	1.35	2	0.25
Green Curve	2.73	0.546	1.13	1.35	2	0.375
Red Curve	2.73	0.546	1.13	1.35	2	0.5

TABLE 10

VALUES OF CONSTANTS RELATED TO MOTOR

So, from the table 9 and table 10, it can be concluded that the control engineering parameters have no effect on both of the constants rather the effect of changing these parameters is the change of mechanical time constant as it is the function of torque constant and back EMF constant. On the other hand, steady state output is directly related to these constants.

3.6 Fuctuating Torque Constant and Back EMF Constant such that Mechanical Time Constant is Grearer than Unity

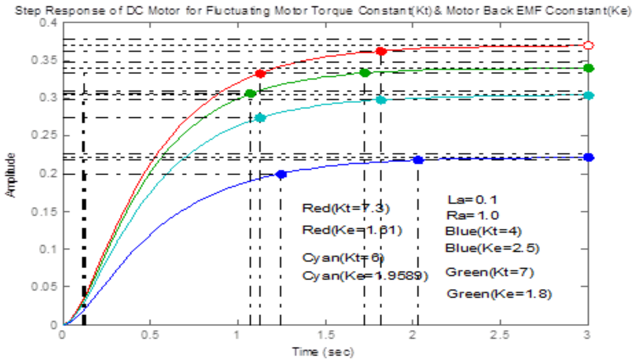


Fig.6. Step response fluctuaing torque constant and back EMF constant

TABLE 11

VALUES OF CONTROLLED PARAMETRS

Obs.	% OS	t_r Rise time (Sec.)	t_p Peak Time (Sec.)	t_s Settling Time (Sec.)	t_{ss} Steady State time (Sec.)	Steady State Output
Blue Curve	0	1.12	>3	2.03	3	0.222
Green Curve	0	1.26	>3.5	2.29	3.5	0.358
Red Curve	0	1.24	>3.5	2.26	3.5	0.395
Cyan Curve	small	1.24	>3.5	2.26	3.5	0.304

TABLE 12

VALUES OF CONSTANTS RELATED TO MOTOR

Obs.	R_a	L_a	t_e	K_t	K_e	D_m	J_m	t_m
Blue Curve	1	0.1	0.1	4	2.5	8	10	1
Green Curve	1	0.1	0.1	5.5	1.5	8	10	1.212
Red	1	0.1	0.1	6.5	1.3	8	10	1.183

Curve								
Cyan Curve	1	0.1	0.1	5	1.7	8	10	1.183

As already explained that there is no effect of mechanical time constant on steady state output and steady state output has proportional effect on torque constant and inverse proportional effect on back EMF constant. So, from the fig.6, table 11 and table 12, it is shown that if the value of torque constant and back EMF constant are changed at same time, the effect is the same as these are changed individually. The effects on rise time, overshoot, peak time and steady state time are same though constants are changed individually.

3.7 Fuctuating Torque Constant and Back EMF Constant such that Mechanical Time Constant is Less than Unity

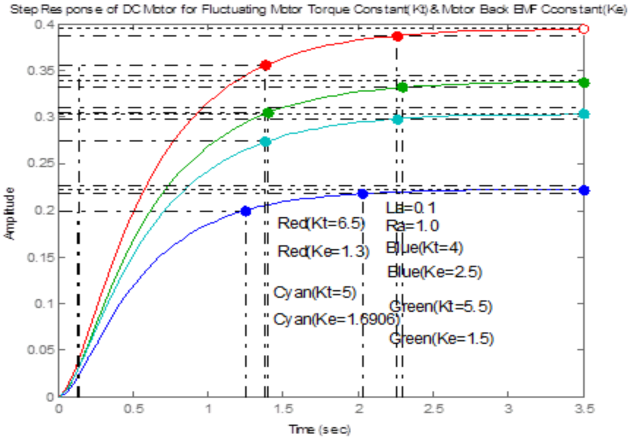


Fig.7. Step response fluctuaing torque constant and back EMF constant

TABLE 13

VALUES OF CONTROLLED PARAMETRS

Obs.	% OS	t_r Rise time (Sec.)	t_p Peak Time (Sec.)	t_s Settling Time (Sec.)	t_{ss} Steady State time (Sec.)	Steady State Output
Blue Curve	0	1.12	>3	2.03	3	0.222
Green Curve	0	0.952	>3	1.72	2.5	0.34
Red Curve	0	1	>3	1.82	2.5	0.37
Cyan Curve	0	1	>3	1.82	3	0.304

TABLE 14

VALUES OF CONSTANTS RELATED TO MOTOR

Obs.	R_a	L_a	t_e	K_t	K_e	D_m	J_m	t_m
Blue Curve	1	0.1	0.1	4	2.5	8	10	1
Green Curve	1	0.1	0.1	7	1.8	8	10	0.794
Red Curve	1	0.1	0.1	7.3	1.61	8	10	0.851
Cyan Curve	1	0.1	0.1	6	1.959	8	10	0.851

In this case also steady state output is proportional to the torque constant and inverse proportional to back EMF constant though the mechanical time constant is less than unity. So, mechanical time constant has no effect on steady state output rather than torque constant and back EMF constant.

On the contrary, when mechanical time constant is changed under unity, rise time, peak time, steady state time and settling time are changed similarly as explained before. But there is no change in these parameters when there is no change in mechanical time constant.

3.8 Varying Equivalent Viscous Damping Coefficient (Dm)

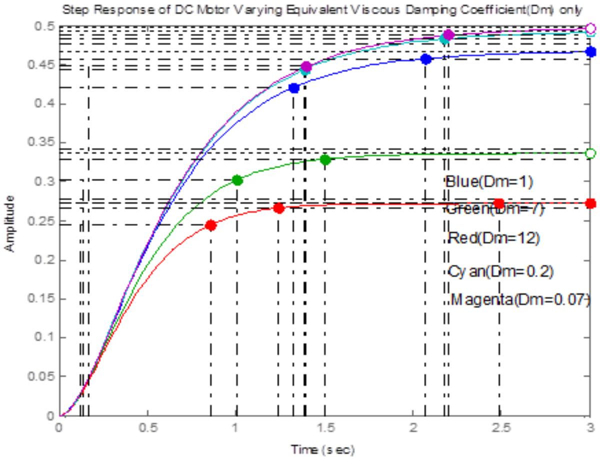


Fig.8. Step response varying equivalent viscous damping coefficient

TABLE 15

VALUES OF CONTROLLED PARAMETRS

Obs.	% OS	t_r Rise time (Sec.)	t_p Peak Time (Sec.)	t_s Settling Time (Sec.)	t_{ss} Steady State time (Sec.)	Steady State Output
Blue Curve	Very small	1.16	>3	2.07	3	0.467
Green Curve	0	0.867	>3	1.5	2.5	0.336
Red Curve	0.004	0.733	2.48	1.24	1.8	0.272
Cyan Curve	Very small	1.22	>3	2.18	3	0.493
Magneta Curve	Very small	1.23	>3	2.2	3	0.498

TABLE 16

VALUES OF CONSTANTS RELATED TO MOTOR

Obs.	R_a	L_a	t_e	K_t	K_e	D_m	J_m	t_m
Blue Curve	0.7	0.1	0.143	5	2	1	10	0.7
Green Curve	0.7	0.1	0.143	5	2	7	10	0.7
Red Curve	0.7	0.1	0.143	5	2	12	10	0.7
Cyan Curve	0.7	0.1	0.143	5	2	0.2	10	0.7
Magneta Curve	0.7	0.1	0.143	5	2	0.07	10	0.7

From the fig.8, table 15 and table 16, it is shown that if equivalent viscous damping coefficient is increased, overshoot becomes zero with fluctuation of its magnitude. So, one peak of overshoot is greater than the next successive one and thus overshoot becomes zero. That means for excessive viscous friction, motor will not rotate.

Again, if equivalent viscous damping coefficient is decreased, overshoot becomes zero with fluctuation of its magnitude. So, one peak of overshoot is greater than the next successive one and thus overshoot becomes zero.

With increasing equivalent viscous damping coefficient, rise time and settling time are decreased but steady state time is increased. If equivalent viscous damping coefficient is decreased, speed of motor reaches at maximum possible stage and then speed is not changed with decreasing equivalent viscous damping coefficient.

Peak time becomes minimum for a particular value of equivalent viscous damping coefficient depending on the size and gear of motor. After that if we increase or decrease the value of equivalent viscous damping coefficient, peak time is increased.

3.9 Varying Equivalent Inertia Coefficient (Jm)

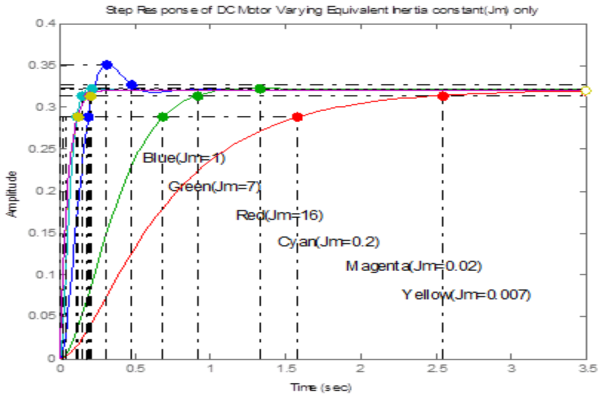


Fig.9. Step response varying equivalent inertia coefficient

TABLE 17

VALUES OF CONTROLLED PARAMETRS

Obs.	% OS	t_r(Sec.) Rise time (Sec.)	t_p Peak Time (Sec.)	t_s Settling Time (Sec.)	t_{ss} Steady State time (Sec.)	Steady State Output
Blue Curve	9.44	0.149	0.311	0.476	0.7	0.321
Green Curve	0.473	0.576	1.33	0.913	1.5	0.321
Red Curve	0	1.4	>3.5	2.54	3.5	0.321
Cyan Curve	0.75	.094	0.207	0.147	3.5	0.321
Magenta Curve	0	0.109	>3.5	0.197	3.5	0.321
Yellow Curve	Very small	0.149	>3.5	0.199	3.5	0.321

TABLE 18

VALUES OF CONSTANTS RELATED TO MOTOR

Obs.	R_a	L_a	t_e	K_t	K_e	D_m	J_m	t_m
Blue Curve	0.7	0.1	0.143	5	2	8	1	0.07
Green Curve	0.7	0.1	0.143	5	2	8	7	0.49
Red Curve	0.7	0.1	0.143	5	2	8	16	1.12
Cyan Curve	0.7	0.1	0.143	5	2	8	0.2	0.014
Magenta Curve	0.7	0.1	0.143	5	2	8	0.02	0.0014
Yellow Curve	0.7	0.1	0.143	5	2	8	0.007	0.0005

From the fig.9, table 17 and table 18, it is observed that, at unity equivalent inertia coefficient, overshoot becomes maximum. But when equivalent inertia coefficient is deviated from unity towards either decreasing or increasing; overshoot becomes zero with fluctuation of its magnitude within small range. So, one peak of overshoot is greater than the next successive one and thus overshoot becomes zero.

Again, if equivalent inertia coefficient is increased from unity, rise time is increased and if equivalent inertia coefficient is decreased from unity, rise time is decreased until a particular value and then it fluctuates within a very small range.

Settling time and peak time become minimum at maximum overshoot and then if equivalent inertia coefficient is increased or decreased, both the time parameters are increased at maximum possible stage. But at maximum overshoot, steady state time is minimum and all other cases cause gradual increase in steady state time.

4 DESIGN CONCLUSION

In conclusion, for design a more perfect DC motor whose characteristics will be suitable for using in very precise works, it should be kept in minds which have been explained throughout the paper. For adjusting the parameters so far explained, resistance should be chosen around middle of unity in accordance with motor size and armature inductance should be kept as low as possible for better response.

Motor torque constant should be kept as moderately high as possible in accordance with the size of motor and better suited level of motor back EMF constant's value is moderately low. Choice of equivalent viscous damping coefficient is better when its range is towards as maximum low as possible. Besides, low level but towards unity is better choice of motor inertia constant.

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